PAD FOOTING ANALYSIS AND DESIGN (BS8110-1:1997)

Pad footing details
Length of pad footing; \( L = 2500 \) mm
Width of pad footing; \( B = 1500 \) mm
Area of pad footing; \( A = L \times B = 3750 \) m²
Depth of pad footing; \( h = 400 \) mm
Depth of soil over pad footing; \( h_{\text{soil}} = 200 \) mm
Density of concrete; \( \rho_{\text{conc}} = 23.6 \) kN/m³

Column details
Column base length; \( l_A = 300 \) mm
Column base width; \( b_A = 300 \) mm
Column eccentricity in \( x \); \( e_{PA} = 0 \) mm
Column eccentricity in \( y \); \( e_{PY} = 0 \) mm

Soil details
Dense, moderately graded, sub-angular, gravel
Mobilisation factor; \( m = 1.5 \)
Density of soil; \( \rho_{\text{soil}} = 20.0 \) kN/m³
Design shear strength; \( \phi' = 25.0 \) deg
Design base friction; \( \delta = 19.3 \) deg
Allowable bearing pressure; \( P_{\text{bearing}} = 200 \) kN/m²

Axial loading on column
Dead axial load on column; \( P_{GA} = 200.0 \) kN
### Project
Pad footing analysis and design (BS8110-1:1997)

### Section
Civil & Geotechnical Engineering

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### Imposed axial load on column:
- \( P_{QA} = 165.0 \text{ kN} \)
- \( P_{WA} = 0.0 \text{ kN} \)
- \( P_A = 365.0 \text{ kN} \)

### Foundation loads
- Dead surcharge load:
  \( F_{Gsur} = 0.000 \text{ kN/m}^2 \)
- Imposed surcharge load:
  \( F_{Qsur} = 0.000 \text{ kN/m}^2 \)
- Pad footing self weight:
  \( F_{swt} = h \times \rho_{conc} = 9.440 \text{ kN/m}^2 \)
- Soil self weight:
  \( F_{soil} = h_{soil} \times \rho_{soil} = 4.000 \text{ kN/m}^2 \)
- Total foundation load:
  \( F = A \times (F_{Gsur} + F_{Qsur} + F_{swt} + F_{soil}) = 50.4 \text{ kN} \)

### Horizontal loading on column base
- Dead horizontal load in x direction:
  \( H_{GxA} = 20.0 \text{ kN} \)
- Imposed horizontal load in x direction:
  \( H_{QxA} = 15.0 \text{ kN} \)
- Wind horizontal load in x direction:
  \( H_{WxA} = 0.0 \text{ kN} \)
- Total horizontal load in x direction:
  \( H_x = 35.0 \text{ kN} \)
- Dead horizontal load in y direction:
  \( H_{GyA} = 5.0 \text{ kN} \)
- Imposed horizontal load in y direction:
  \( H_{QyA} = 5.0 \text{ kN} \)
- Wind horizontal load in y direction:
  \( H_{WyA} = 0.0 \text{ kN} \)
- Total horizontal load in y direction:
  \( H_y = 10.0 \text{ kN} \)

### Moment on column base
- Dead moment on column in x direction:
  \( M_{GxA} = 15.000 \text{ kNm} \)
- Imposed moment on column in x direction:
  \( M_{QxA} = 10.000 \text{ kNm} \)
- Wind moment on column in x direction:
  \( M_{WxA} = 0.000 \text{ kNm} \)
- Total moment on column in x direction:
  \( M_x = 25.000 \text{ kNm} \)
- Dead moment on column in y direction:
  \( M_{GyA} = 25.000 \text{ kNm} \)
- Imposed moment on column in y direction:
  \( M_{QyA} = 30.000 \text{ kNm} \)
- Wind moment on column in y direction:
  \( M_{WyA} = 0.000 \text{ kNm} \)
- Total moment on column in y direction:
  \( M_y = 55.000 \text{ kNm} \)

### Check stability against sliding
- Resistance to sliding due to base friction:
  \( H_{friction} = \max([P_{GA} + (F_{Gsur} + F_{swt} + F_{soil}) \times A], 0 \text{ kN}) \times \tan(\delta) = 87.7 \text{ kN} \)
- Passive pressure coefficient:
  \( K_p = (1 + \sin(\phi')) / (1 - \sin(\phi')) = 2.464 \)
- Passive resistance of soil in x direction:
  \( H_{xpas} = 0.5 \times K_p \times (h^2 + 2 \times h \times h_{soil}) \times B \times \rho_{soil} = 11.8 \text{ kN} \)
- Total resistance to sliding in x direction:
  \( H_{xres} = H_{friction} + H_{xpas} = 99.5 \text{ kN} \)
  **PASS - Resistance to sliding is greater than horizontal load in x direction**

- Passive resistance of soil in y direction:
  \( H_{ypas} = 0.5 \times K_p \times (h^2 + 2 \times h \times h_{soil}) \times L \times \rho_{soil} = 19.7 \text{ kN} \)
- Total resistance to sliding in y direction:
  \( H_{yres} = H_{friction} + H_{ypas} = 107.4 \text{ kN} \)**
Check stability against overturning in x direction

Total overturning moment:

\[ M_{\text{OT}} = M_{\text{fa}} + H_{\text{fa}} \times h = 39.000 \text{ kNm} \]

Restoring moment in x direction

Foundation loading:

\[ M_{\text{fusr}} = A \times (F_{\text{gsur}} + F_{\text{swt}} + F_{\text{soil}}) \times L / 2 = 63.000 \text{ kNm} \]

Axial loading on column:

\[ M_{\text{axial}} = (P_{\text{G}}A) \times (L / 2 - e_{\text{PxA}}) = 250.000 \text{ kNm} \]

Total restoring moment:

\[ M_{\text{res}} = M_{\text{fusr}} + M_{\text{axial}} = 313.000 \text{ kNm} \]

PASS - Restoring moment is greater than overturning moment in x direction

Check stability against overturning in y direction

Total overturning moment:

\[ M_{\text{OT}} = M_{\text{ya}} + H_{\text{ya}} \times h = 59.000 \text{ kNm} \]

Restoring moment in y direction

Foundation loading:

\[ M_{\text{ysur}} = A \times (F_{\text{gsur}} + F_{\text{swt}} + F_{\text{soil}}) \times B / 2 = 37.800 \text{ kNm} \]

Axial loading on column:

\[ M_{\text{yaxial}} = (P_{\text{G}}A) \times (B / 2 - e_{\text{PyA}}) = 150.000 \text{ kNm} \]

Total restoring moment:

\[ M_{\text{res}} = M_{\text{ysur}} + M_{\text{yaxial}} = 187.800 \text{ kNm} \]

PASS - Restoring moment is greater than overturning moment in y direction

Calculate pad base reaction

Total base reaction:

\[ T = F + P_{\text{A}} = 415.4 \text{ kN} \]

Eccentricity of base reaction in x:

\[ e_{\text{Tx}} = (P_{\text{A}} \times e_{\text{PxA}} + M_{\text{fa}} + H_{\text{fa}} \times h) / T = 94 \text{ mm} \]

Eccentricity of base reaction in y:

\[ e_{\text{Ty}} = (P_{\text{A}} \times e_{\text{PyA}} + M_{\text{ya}} + H_{\text{ya}} \times h) / T = 142 \text{ mm} \]

Check pad base reaction eccentricity

\[ \text{abs}(e_{\text{Tx}}) / L + \text{abs}(e_{\text{Ty}}) / B = 0.132 \]

Base reaction acts within combined middle third of base

Calculate pad base pressures

Total base reaction:

\[ T = F + P_{\text{A}} = 415.4 \text{ kN} \]

Minimum base pressure:

\[ q_{\text{min}} = \text{min}(q_1, q_2, q_3, q_4) = 22.880 \text{ kN/m}^2 \]

Maximum base pressure:

\[ q_{\text{max}} = \text{max}(q_1, q_2, q_3, q_4) = 198.667 \text{ kN/m}^2 \]

PASS - Maximum base pressure is less than allowable bearing pressure
Partial safety factors for loads

Partial safety factor for dead loads;  \( \gamma_{fG} = 1.40 \)
Partial safety factor for imposed loads;  \( \gamma_{fQ} = 1.60 \)
Partial safety factor for wind loads;  \( \gamma_{fW} = 0.00 \)

Ultimate axial loading on column
Ultimate axial load on column;  \( P_{uA} = P_{GA} \times \gamma_{fG} + P_{QA} \times \gamma_{fQ} + P_{WA} \times \gamma_{fW} = 544.0 \) kN

Ultimate foundation loads
Ultimate foundation load;  \( F_u = A \times ([F_{Gsur} + F_{Qsur} + F_{swt} + F_{soil}] \times \gamma_{fG} + F_{Qsur} \times \gamma_{fG}) = 70.6 \) kN

Ultimate horizontal loading on column
Ultimate horizontal load in x direction;  \( H_{xUA} = H_{GxA} \times \gamma_{fG} + H_{QxA} \times \gamma_{fQ} + H_{WxA} \times \gamma_{W} = 52.0 \) kN
Ultimate horizontal load in y direction;  \( H_{yUA} = H_{Gya} \times \gamma_{fG} + H_{Qya} \times \gamma_{fQ} + H_{Wya} \times \gamma_{W} = 15.0 \) kN

Ultimate moment on column
Ultimate moment on column in x direction;  \( M_{xA} = M_{GxA} \times \gamma_{fG} + M_{QxA} \times \gamma_{fQ} + M_{WxA} \times \gamma_{W} = 37.000 \) kNm
Ultimate moment on column in y direction;  \( M_{yA} = M_{Gya} \times \gamma_{fG} + M_{Qya} \times \gamma_{fQ} + M_{Wya} \times \gamma_{W} = 83.000 \) kNm
**Calculate ultimate pad base reaction**

Ultimate base reaction:

\[ T_u = F_u + P_{uA} = 614.6 \text{ kN} \]

Eccentricity of ultimate base reaction in x:

\[ e_{TuA} = \frac{(P_{uA} \times e_{PA} + M_{uA} + H_{uA} \times h)}{T_u} = 94 \text{ mm} \]

Eccentricity of ultimate base reaction in y:

\[ e_{Tyu} = \frac{(P_{uA} \times e_{PYA} + M_{uA} + H_{yuA} \times h)}{T_u} = 145 \text{ mm} \]

**Calculate ultimate pad base pressures**

- **31.957 kN/m²**
  \[ q_{1u} = T_u/A - 6 \times T_u \times e_{TuA}/(L \times A) - 6 \times T_u \times e_{Tyu}/(B \times A) = \]

- **221.824 kN/m²**
  \[ q_{2u} = T_u/A - 6 \times T_u \times e_{TuA}/(L \times A) + 6 \times T_u \times e_{Tyu}/(B \times A) = \]

- **105.941 kN/m²**
  \[ q_{3u} = T_u/A + 6 \times T_u \times e_{TuA}/(L \times A) - 6 \times T_u \times e_{Tyu}/(B \times A) = \]

- **295.808 kN/m²**
  \[ q_{4u} = T_u/A + 6 \times T_u \times e_{TuA}/(L \times A) + 6 \times T_u \times e_{Tyu}/(B \times A) = \]

**Calculate rate of change of base pressure in x direction**

Left hand base reaction:

\[ f_{L} = (q_{1u} + q_{2u}) \times B / 2 = 190.336 \text{ kN/m} \]

Right hand base reaction:

\[ f_{R} = (q_{3u} + q_{4u}) \times B / 2 = 301.312 \text{ kN/m} \]

Length of base reaction:

\[ L_x = L = 2500 \text{ mm} \]

Rate of change of base pressure:

\[ C_x = (f_{R} - f_{L}) / L_x = 44.390 \text{ kN/m/m} \]

**Calculate pad lengths in x direction**

Left hand length:

\[ L_L = L / 2 + e_{PA} = 1250 \text{ mm} \]

Right hand length:

\[ L_R = L / 2 - e_{PA} = 1250 \text{ mm} \]

**Calculate ultimate moments in x direction**

Ultimate moment in x direction:

\[ M_x = f_{L} = L_x^2/2 + C_x \times L_x^2/6 + F_0 \times L_x^2/2 \times L + H_{uA} \times h + M_{uA} = 198.900 \text{ kNm} \]

**Calculate rate of change of base pressure in y direction**

Top edge base reaction:

\[ f_{uT} = (q_{2u} + q_{4u}) \times L / 2 = 647.040 \text{ kN/m} \]

Bottom edge base reaction:

\[ f_{uB} = (q_{1u} + q_{3u}) \times L / 2 = 172.373 \text{ kN/m} \]

Length of base reaction:

\[ L_y = B = 1500 \text{ mm} \]

Rate of change of base pressure:

\[ C_y = (f_{uB} - f_{uT}) / L_y = -316.444 \text{ kN/m/m} \]

**Calculate pad lengths in y direction**

Top length:

\[ L_T = B / 2 - e_{PYA} = 750 \text{ mm} \]

Bottom length:

\[ L_B = B / 2 + e_{PYA} = 750 \text{ mm} \]

**Calculate ultimate moments in y direction**

Ultimate moment in y direction:

\[ M_y = f_{uT} \times L_T^2/2 + C_y \times L_T^2/6 - F_0 \times L_T^2/2 \times B = 146.500 \text{ kNm} \]

**Material details**

Characteristic strength of concrete:

\[ f_{cu} = 30 \text{ N/mm}^2 \]
Characteristic strength of reinforcement; \( f_y = 500 \text{ N/mm}^2 \)
Characteristic strength of shear reinforcement; \( f_{yv} = 500 \text{ N/mm}^2 \)
Nominal cover to reinforcement; \( c_{nom} = 30 \text{ mm} \)

**Moment design in x direction**

Diameter of tension reinforcement; \( \phi_{xB} = 12 \text{ mm} \)
Depth of tension reinforcement; \( d_x = h - c_{nom} - \phi_{xB} / 2 = 364 \text{ mm} \)

Design formula for rectangular beams (cl 3.4.4.4)

\[
K_x = \frac{M_x}{B \times d_x^2 \times f_{cu}} = 0.033
\]
\[
K'_x = 0.156
\]

\( K_x < K'_x \) compression reinforcement is not required

Lever arm; \( z_x = d_x \times \min((0.5 + \sqrt{0.25 - K_x / 0.9}), 0.95) = 346 \text{ mm} \)

Area of tension reinforcement required; \( A_{s,x_{req}} = M_x / (0.87 \times f_y \times z_x) = 1322 \text{ mm}^2 \)
Minimum area of tension reinforcement; \( A_{s,x_{min}} = 0.0013 \times B \times h = 780 \text{ mm}^2 \)
Tension reinforcement provided; 12 No. 12 dia. bars bottom (125 centres)
Area of tension reinforcement provided; \( A_{s,x_{B_prov}} = N_{xB} \times \pi \times \phi_{xB}^2 / 4 = 1357 \text{ mm}^2 \)

**PASS - Tension reinforcement provided exceeds tension reinforcement required**

**Moment design in y direction**

Diameter of tension reinforcement; \( \phi_{yB} = 12 \text{ mm} \)
Depth of tension reinforcement; \( d_y = h - c_{nom} - \phi_{xB} - \phi_{yB} / 2 = 352 \text{ mm} \)

Design formula for rectangular beams (cl 3.4.4.4)

\[
K_y = \frac{M_y}{L \times d_y^2 \times f_{cu}} = 0.016
\]
\[
K'_y = 0.156
\]

\( K_y < K'_y \) compression reinforcement is not required

Lever arm; \( z_y = d_y \times \min((0.5 + \sqrt{0.25 - K_y / 0.9}), 0.95) = 334 \text{ mm} \)

Area of tension reinforcement required; \( A_{s,y_{req}} = M_y / (0.87 \times f_y \times z_y) = 1007 \text{ mm}^2 \)
Minimum area of tension reinforcement; \( A_{s,y_{min}} = 0.0013 \times L \times h = 1300 \text{ mm}^2 \)
Tension reinforcement provided; 13 No. 12 dia. bars bottom (200 centres)
Area of tension reinforcement provided; \( A_{s,y_{B_prov}} = N_{yB} \times \pi \times \phi_{yB}^2 / 4 = 1470 \text{ mm}^2 \)

**PASS - Tension reinforcement provided exceeds tension reinforcement required**

Calculate ultimate shear force at d from right face of column

Ultimate pressure for shear; \( q_{su} = (q_{tu} + C_x \times (L / 2 + \varepsilon_{PA} + I_1 / 2 + d_x) / B + q_{cu}) / 2 \)
\( q_{su} = 189.984 \text{ kN/m}^2 \)

Area loaded for shear; \( A_s = B \times \min(3 \times (L / 2 - \varepsilon_{PA}), L / 2 - \varepsilon_{PA} - I_1 / 2 - d) \)
\( = 1.104 \text{ m}^2 \)

Ultimate shear force; \( V_{su} = A_s \times (q_{su} - F_u / A) = 188.970 \text{ kN} \)

Shear stresses at d from right face of column (cl 3.5.5.2)

Design shear stress; \( v_{su} = V_{su} / (B \times d_x) = 0.346 \text{ N/mm}^2 \)
From BS 8110:Part 1:1997 - Table 3.8

Design concrete shear stress;  
\[ v_c = 0.432 \text{ N/mm}^2 \]

Allowable design shear stress;  
\[ v_{\text{max}} = \min(0.8N/\text{mm}^2 \times \sqrt{(f_{\text{cu}} / 1 \text{ N/mm}^2)}, 5 \text{ N/mm}^2) \]
\[ = 4.382 \text{ N/mm}^2 \]

**PASS -**  \( v_{pu} < v_c \) - No shear reinforcement required

Calculate ultimate punching shear force at face of column

Ultimate pressure for punching shear;  
\[ q_{pu} = q_{tu} + \left[ \frac{(L/2+e_{PA}-l/2)}{C_y}\right] 2 \times B - \left[ \frac{(B/2+e_{PA})}{C_y}\right] L \]
\[ q_{pu} = 163.883 \text{ kN/m}^2 \]

Average effective depth of reinforcement;  
\[ d = \frac{d_x + d_y}{2} = 358 \text{ mm} \]

Area loaded for punching shear at column;  
\[ A_{PA} = (l_x)(b_x) = 0.090 \text{ m}^2 \]

Length of punching shear perimeter;  
\[ u_{PA} = 2 \times (l_x) + 2 \times (b_x) = 1200 \text{ mm} \]

Ultimate shear force at shear perimeter;  
\[ V_{pu} = P_{u} / A - q_{pu} \times A_{PA} = 530.944 \text{ kN} \]

Effective shear force at shear perimeter;  
\[ V_{pu,\text{eff}} = V_{pu} \times 1.25 = 663.678 \text{ kN} \]

Punching shear stresses at face of column (cl 3.7.7.2)

Design shear stress;  
\[ V_{pu} = V_{pu,\text{eff}} / (u_{PA} \times d) = 2.633 \text{ N/mm}^2 \]

Allowable design shear stress;  
\[ v_{\text{max}} = \min(0.8N/\text{mm}^2 \times \sqrt{(f_{\text{cu}} / 1 \text{ N/mm}^2)}, 5 \text{ N/mm}^2) \]
\[ = 4.382 \text{ N/mm}^2 \]

**PASS - Design shear stress is less than allowable design shear stress**

Calculate ultimate punching shear force at perimeter of 1.5 d from face of column

Ultimate pressure for punching shear;  
\[ q_{pu,1.5d} = q_{tu} + \left[ \frac{(L/2+e_{PA}-l/2)}{C_y}\right] 2 \times B - \left[ \frac{(B/2+e_{PA})}{C_y}\right] L \]
\[ q_{pu,1.5d} = 163.883 \text{ kN/m}^2 \]

Average effective depth of reinforcement;  
\[ d = \frac{d_x + d_y}{2} = 358 \text{ mm} \]

Area loaded for punching shear at column;  
\[ A_{PA,1.5d} = (l_x+2 \times 1.5 \times d/2) \times B - \left[ \frac{(B/2+e_{PA})}{C_y}\right] L \]
\[ A_{PA,1.5d} = 2.061 \text{ m}^2 \]

Length of punching shear perimeter;  
\[ u_{PA,1.5d} = 2 \times B = 3000 \text{ mm} \]

Ultimate shear force at shear perimeter;  
\[ V_{pu,1.5d} = P_{u} / A - q_{pu,1.5d} \times A_{PA,1.5d} = 245.018 \text{ kN} \]

Effective shear force at shear perimeter;  
\[ V_{pu,1.5d,\text{eff}} = V_{pu,1.5d} \times 1.25 = 306.272 \text{ kN} \]

Punching shear stresses at perimeter of 1.5 d from face of column (cl 3.7.7.2)

Design shear stress;  
\[ V_{pu,1.5d} = V_{pu,1.5d,\text{eff}} / (u_{PA,1.5d} \times d) = 0.285 \text{ N/mm}^2 \]

From BS 8110:Part 1:1997 - Table 3.8

Design concrete shear stress;  
\[ v_c = 0.409 \text{ N/mm}^2 \]

Allowable design shear stress;  
\[ v_{\text{max}} = \min(0.8N/\text{mm}^2 \times \sqrt{(f_{\text{cu}} / 1 \text{ N/mm}^2)}, 5 \text{ N/mm}^2) \]
\[ = 4.382 \text{ N/mm}^2 \]

**PASS -**  \( v_{pu,1.5d} < v_c \) - No shear reinforcement required
Shear at d from column face

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Punching shear perimeter at 1.5 × d from column face

13 No. 12 dia. bars btm (200 c/c)

12 No. 12 dia. bars btm (125 c/c)